ISSN 2395-1621

Modal analysis of rotor assembly of vertical turbine pump

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ABSTRACT

Vibration is an undesirable behaviour of dynamic systems which is generated due to resonance, imbalance and other factors. Here, a case of a Vertical Turbine (VT) pump is considered subjected to high levels of vibration amplitudes if the forcing frequency approaches any of the system's natural frequency within a range of its ±20%. Earlier, the pump is known to have resonance problems after being installed on-site. The pump structure resonates at operating speed and hence is subjected to large displacements. The reason for this was that, these pumps were not validated regarding their response in the working environment. As a result, more time and money were needed to be poured in for correcting the design. To avoid such instances, it is necessary to carry out a modal analysis of the pump to determine its dynamic characteristics such as its natural frequencies and corresponding mode shapes. This will limit its vibration amplitudes within the values specified in ISO 10816. For this, we have analyzed the rotor assembly of VT Pump theoretically, numerically and experimentally. The system is modelled as a lumped mass structure to theoretically determine its torsional natural frequencies and as continuous system to determine its transverse natural frequencies. The equations of motion for the system are solved using MATLAB. A three dimensional model of the system is analyzed using ANSYS Workbench. The numerical model is validated with the results of the theoretical analysis. Validated numerical model is used to determine the dynamic characteristics of the system with realistic boundary conditions. This model is validated with bump test experimentation on the actual system. Vibration amplitudes also were within the specified limits. This validated the working of the pump to be safe in the working environment.

Keywords— **bump test, modal analysis, natural frequencies, numerical analysis, theoretical modelling, Vertical Turbine pump.**

ARTICLE INFO

Article History

Received :18th November 2015

Received in revised form :

19th November 2015

Accepted : 21st November , 2015

Published online :

22nd November 2015

I. INTRODUCTION

Vibration is a well-known undesirable behaviour of dynamical systems which generates noise and cause fatigue, which initiates cracks in mechanical structures. The project is regarding a Vertical Turbine (VT) pump in a pump industry. The product is used in applications such as Drinking water supply schemes, industrial plants and power plants for cooling water supply, irrigation and many others where large quantity of water is to be transported.

The VT pump consists of a motor (594 rpm; i.e. motor forcing frequency = 9.9 Hz), motor stool, delivery bend, thrust bearing, column pipe, impeller, diffuser, shaft, etc. The construction of the pump can be viewed in figure 1.

The column pipes along with the MS and DB encompass the rotor assembly of the pump.

In some earlier cases, the pump is known to have resonance problems after being installed on-site. Some of the natural frequencies of the system lie within ±20% of its forcing frequency. Hence, the pump structure resonates at operating speed and hence is subjected to large

displacements. As a result, the Company has to put in more efforts in terms of time and money for correcting the design. This affects the productivity and efficiency of the Company in delivering their products. To avoid such incidents any further, the Company desires to carry out a modal analysis of the pump to limit its vibration amplitudes within the values specified in the standard 10816.

The pump can be modelled into an analytical model and a theoretical analysis can be carried out on this model followed by numerical and experimental analyses for the same. However, analysing such massive assembly theoretically is quite difficult and time consuming. Modelling all the boundary conditions of the complete assembly would complicate the process of deriving the equations of motion for the system.

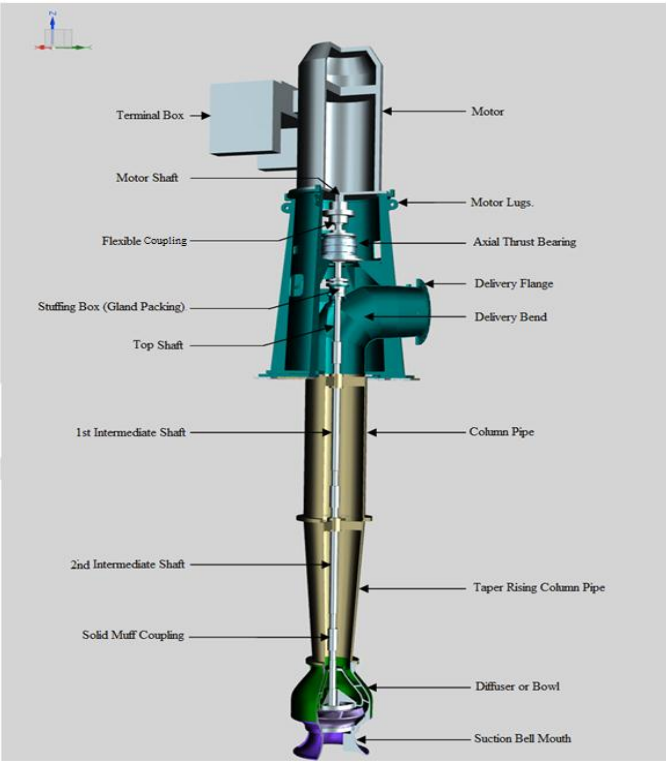


Fig.1 Cross Section of Vertical Turbine Pump

Hence, to simplify the system only the moving parts (rotor assembly) will be analyzed theoretically, numerically and experimentally. The rest of the assembly shall imply different boundary conditions on the rotor assembly.

A. Objectives

In order to achieve the goal of limiting the vibration amplitudes in the rotor assembly, it is required to fulfil the following objectives.

1. To prepare analytical model of the rotor assembly as a torsional spring-mass system and as a continuous cantilever shaft with mass at free end; calculate natural frequencies analytically
2. To obtain the corresponding mode shapes.
3. To prepare three dimensional model of the assembly and to analyze it numerically
4. To derive response of the system experimentally.
5. To modify the design in case of occurrence of resonance.
6. To acquire the vibration levels of the rotor and shafts.

B. Methodology

For fulfilling the aforementioned objectives, a methodology needs to be followed described as follows. The project will be completed in three phases, viz; analytical, numerical and experimental.

1) Analytical:
   a) Analytical modelling of the system as an undamped torsional lumped spring-mass system using Newton's second law of motion.
   b) Creating a MATLAB program using Holzer's method and extracting the torsional natural frequency values and their corresponding mode shapes.
   c) Analytical modelling of the system as a cantilever with mass attached at the free end for deriving the transverse natural frequencies.
   d) Creating a MATLAB program for solving the frequency equation of the system derived by solving the equations of motion of the system and extracting the transverse natural frequency values and their corresponding mode shapes.
2) Numerical:
   a) Modelling the system in three dimensions using UniGraphics NX 8.5 software.
   b) Generating a solid mesh on the system and post processing using ANSYS 15 software.
   c) If any of the system's natural frequencies coincide within ±20% of the forcing frequency, then, redesigning the part with maximum amplitude at that particular frequency. The part can made more or less stiffer in order to alter that particular frequency.
   d) Performing the same procedure until all the natural frequencies of the system lie ±20% away from the forcing frequency (i.e. not within 7.92 Hz and 11.88 Hz).
3) Experimental:
   a) In experimentation, extracting natural frequencies and corresponding mode shapes using FFT analyzer, as per standard ISO 10816-3.
   b) Performing the FFT testing in both rest and working conditions.
   c) Comparing the results obtained in the three phases, viz; analytical, numerical and experimental.

II. LITERATURE REVIEW

Tony DeMatteo [1] has illustrated the typical steps required to solve resonance problems. The paper explains experimental analysis carried out on two-vane, vertical centrifugal sewage pumps using impact hammer. Also, the paper explains the use of Operational Deflection Shape (ODS) and Modal as powerful tools to understand the sources of vibration. Finite Element Analysis (FEA) is used to model the pump structure and evaluate modifications that move the natural frequencies away from forced vibrations– thus eliminating resonance.

Qingyu Wang et al. [2] identified major sources of uncertainty in torsional modelling. Some degree of uncertainty is always present within the analytical data, the modelling techniques, and the assumptions for excitation and damping. The effect of variation in mass-elastic data is examined, and a comparison between measured and

predicted torsional natural frequencies for numerous cases is presented.

R. E. Cornman [3] has explained the on-site experimentation of vertical water circulating pump using impact testing and vibration measurement. The author also explains the FEA of the structure using shell mesh. A comparison of Field Measurements with ANSYS Measurement is made to check the accuracy obtained. Also, natural frequency in wet condition is determined. The results obtained from FE analysis predicted the natural frequencies within 4% of those obtained experimentally.

D. Smith and G. Woodward [4] discussed the case of excessive wear of impeller, impeller wear rings and seals in a cooking water supply vertical pump. The cause of vibration was the operating frequency being close to pump-motor system mechanical natural frequency. The paper considers the effect of modifying the stiffness of the pump mountings. Thus, the paper directs different ways to shift the natural frequencies of a system. A successful solution was obtained by increasing the stiffness of the pump mountings.

J. Corley [5] presented an analytical and experimental study of a 3000 hp crude oil loading pump. The study shows the large number of possible vibrational modes which can exist near the operational speed of a typical system and the factors which should be considered in predicting resonant frequencies. The paper states that establishing amplitude limits which will assure trouble-free operation and a better definition as to where these measurements should be made are needed to be improved.

M. Corbo et al. [6] put forth the vertical pump with bearing and seal failures. The paper discusses the vibration measurement of the shaft and the column. The paper deals with elimination of the problem of sub-synchronous whirling at approximately one half the running speed. The paper concludes that lateral rotodynamic analysis is a must in the design process of any vertical pump.

T. Fesse and P. Grazier [7] discussed six case studies which illustrate actual problems that encountered during field balancing of different types of rotating machinery. The paper focuses mainly on whirling of rotating shafts and corrective steps needed thereafter. The paper discusses six different case studies and finds the way to connect the rotating parts in the same angular orientation.

Thus, from the above studied reference papers it is seen that, an analytical method such as Holzer's method can be used to predict the natural frequencies of the rotor assembly. Vertically hanging systems are seen to have transverse bending modes. The validations of any system through different methods have errors due to non-linearities and inability to perfectly model the system as per real life conditions.

III. THEORETICAL ANALYSIS

An analytical method can be used to predict the torsional natural frequencies of a system [2]. Accordingly, for the rotor assembly of the VT pump, the system needs to be modelled by considering it as a cantilever structure comprised of lumped masses attached with springs or as a continuous mass cantilever with a mass attached to its free end. The rotor assembly is free to rotate along the axis of the shafts and hence has torsional frequencies. The torsional natural frequencies of the system are derived in the first section. Also, the system can vibrate in transverse direction and so its transverse natural frequencies are derived in the second section.

A. Torsional Theoretical Analysis

To determine these torsional natural frequencies and their corresponding mode shapes Holzer's method is chosen [2]. Here the system is modelled as six discs of moments of inertias J_i that incorporate the masses of the shafts and massless shafts connecting them of torsional stiffness k_{ti}, diagrammatically as shown below,

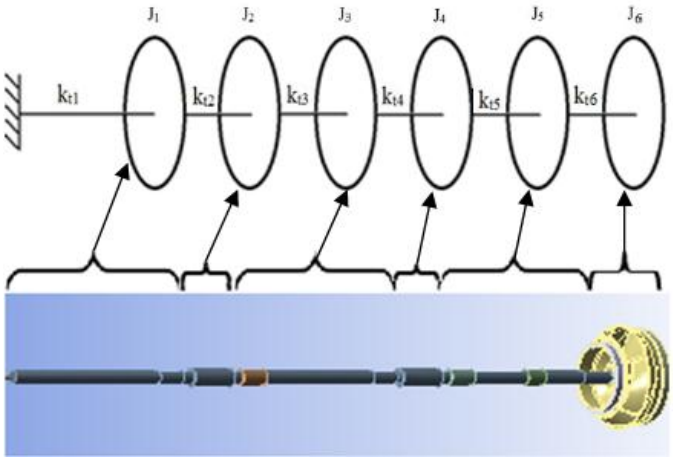


Fig.2 Six rotor model of the rotor assembly

where,

J_i = i^{th} moment of inertia

k_{ti} = i^{th} torsional spring stiffness,

such that, i = 1,2,3,4,5,6

where, 1 = top shaft

2 = 1st solid muff coupling

3 = intermediate shaft

4 = 2nd solid muff coupling

5 = pump shaft

6 = impeller, impeller wear rings, impeller nose cap.

Here,

$$J = \frac{MR^2}{2}$$

where, 'm' is the mass of the component and 'R' is its radial distance from axis of rotation. In case of coupling which is a hollow cylinder,

$J = \frac{M(R_1^2+R_2^2)}{2}$, where, '$R_1$' and '$R_2$' being its inner and outer radii respectively.

$k_t = \frac{GI}{L}$, where 'G' is the modulus of rigidity, 'I' is the polar moment of inertia of the cross-section of the shafts and 'L' is the length of the component.

B. Calculations

The rotor assembly of the VT pump is modelled in 3D modelling software. The volumes of each component are measured and densities of their particular materials are applied. There are two terms for each component to be derived, namely,

1) Moment of inertia of each component (J_i): Since the shafts are of almost constant cross section, these are assumed to be of an average diameter; keeping their volumes and lengths constant. Mass of each component is considered as disc acting at that particular mass's centre of gravity. The volumes (V) and lengths (L) from CAD model are used to determine their respective average diameters (D), masses (M) and moments of inertia (J).

2) Torsional stiffness (k_t) of shafts and couplings: The shear modulus (G) of the shafts of the material ASTM A276 SS410 is calculated as, $E = 2G \cdot (1 + \mu)$

where, E = Modulus of Elasticity = 2,00,000 MPa

μ = Poisson's Ratio = 0.3

Thus,

$$G = 76{,}923.0769 \text{ MPa}$$

The torsional stiffness of the shafts (k_t) as indicated in the mathematical model are calculated by the earlier mentioned formula.

According to Holzer's Method [8] the table is filled with the above data. The method assume the initial approximation of amplitude of vibration, $\beta = 1$. A program for the same is written in MATLAB software for carrying out the iterations. The results found out through the program are summarized as follows:

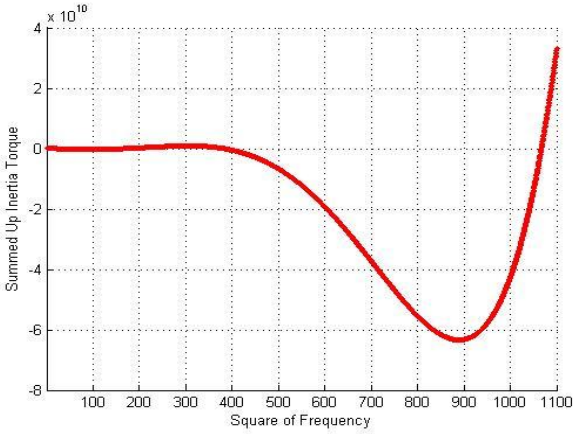


Fig.3 MATLAB Output- Summed Up Inertia Torque vs. Square of Frequency

The values of frequencies at which the corresponding values of summed up inertia torque are zero are identified as the natural frequencies of the system. The values on the X axis are squares of frequencies and hence, their roots are considered to obtain the values of natural frequencies (ω_n).

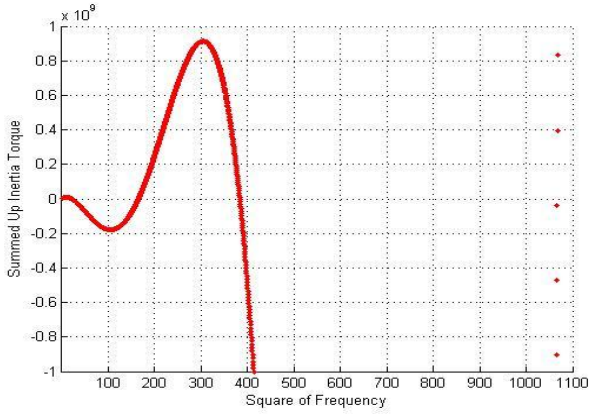


Fig.4 Zoomed View of MATLAB Output- Summed Up Inertia Torque vs. Square of Frequency

The results interpreted from these graphs are summarized as follows,

TABLE I
TORSIONAL THEORETICAL NATURAL FREQUENCIES

Sr. No.	Natural Frequency (Hz)
1	2
2	7
3	10
4	15
5	17

C. *Transverse Theoretical Analysis*

The torsional frequencies of the system have been derived in the previous section. However, such vertically hanging systems are seen to have transverse bending modes also [1],[3]. Hence it is necessary to determine the transverse natural frequencies of the system. The system is considered as a continuous cantilever shaft fixed at one end and attached to a mass at the other end, which is free to move in all directions.

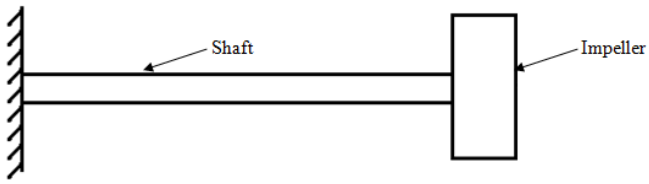


Fig.5 Continuous Mass model of the rotor assembly

The equation of motion for a transverse beam is given as follows [10],

$$c^2 \frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w(x,t)}{\partial t^2} = 0$$

The response of the system is given as,

$$w(x, t) = W(x).T(t)$$

The system is at rest prior to imparting the force. Hence, the response of the system will be a single wave and zero at the start. Thus, considering,

$$T(t) = \sin \omega t$$

Also, reference [10] shows,

$$W(x) = A.\cos(\beta x) + B.\sin(\beta x) + C.\cosh(\beta x) + D.\sinh(\beta x)$$

Since the system is at rest at the start, the displacement and velocity of the system will be zero at the start. The initial conditions can thus be expressed as follows,

a. $w(x, t=0) = w_0(x) = 0$ and

b. $\frac{\partial w(x,t=0)}{\partial t} = \dot{w}_0(x) = 0.$

The system is a vertically hanging shaft fixed at the top and attached to a mass at its bottom. The top end is considered to be fixed and its boundary conditions are,

1) The deflection at the top end, being fixed, is zero.

2) The slope at this end will be zero.

The bottom end is connected to a mass (i.e. at distance x=l from top end) and is free to move in all directions. Here,

1) The bending moment is zero.

2) The shear force balances the resisting force.

Substituting the boundary condition in the above mentioned equation, the frequency obtained is as follows,

$$1+\cos(\beta l).\cosh(\beta l)+q.(\cos(\beta l).\sinh(\beta l)-\sin(\beta l)\cos$$

where, $\beta = \sqrt[4]{\frac{\rho A \omega^2}{EI}}$

$q = \frac{-m\omega^2}{EI\beta^3}$

ρ = Density of the shaft material (kg/mm^3)
A = Area of cross section of shaft (mm^2)
ω = Frequency (Hz)
E = Modulus of Elasticity (MPa)
I = Polar Moment of Inertia of cross-section of shaft (mm^4)
and, m = Mass attached at free end (kg)

The roots of this equation give the natural frequencies of the system. The values of 'ω' for which the value of the above equation equals zero are the natural frequencies of the system.

A program for this is written in MATLAB. The results and graphs of 'Values of Frequency Equation' versus 'Frequency (Hz)' obtained from the program are shown below.

TABLE III
TRANSVERSE THEORETICAL NATURAL FREQUENCIES

Sr. No.	Natural Frequency (Hz)
1	1
2	6
3	13
4	23
5	36
6	50

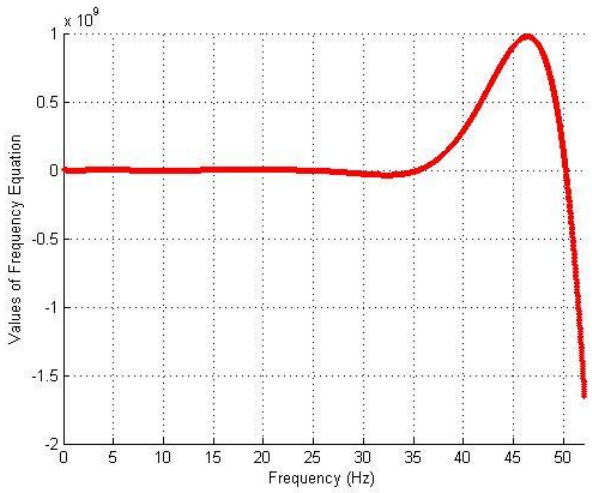


Fig.6 MATLAB Output- Values of Frequency Equation vs. Frequency

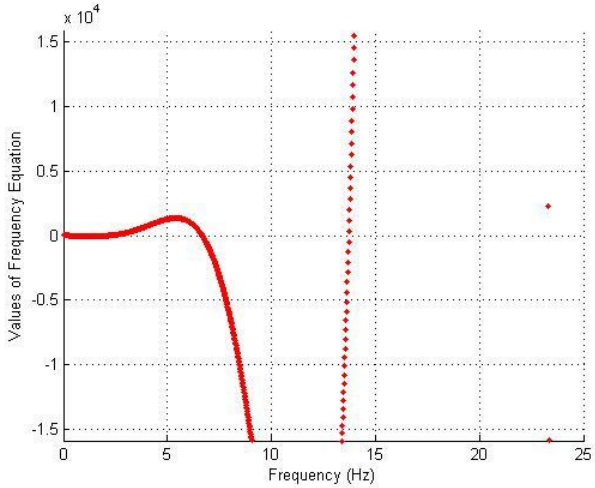


Fig.7 Zoomed View of MATLAB Output- Values of Frequency Equation vs. Frequency

IV. NUMERICAL ANALYSIS

The drafting of the rotor assembly of the VT pump is provided by the company. All parts of the assembly are modelled in three dimensions using UniGraphics NX 8 software. All the modelled parts are assembled together and saved as Parasolid file. A modal analysis is performed on this system using ANSYS Workbench.

A. Validating with Theoretical Analyses

First of all, the model to be used in ANSYS Workbench for numerical analysis needs to be validated with the analytical model. A model as assumed in the theoretical analysis is constructed in Workbench. For validation purpose, results of this system must match with that in theoretical analysis. Once the model to be used in Workbench is validated, constraints and boundary conditions can be applied to the model to make the model as close as possible to the real working condition. The procedure for validating the numerical model is presented as follows,

a) Geometry: The parasolid file of the rotor assembly is imported in Workbench.
b) Connections: All the connections between different parts are set as 'Bonded' which corresponds to welding of parts together.
c) Mesh: A Hex-dominant mesh is generated in ANSYS Workbench software. The mesh size of the shafts is set to 15mm. The Hex-dominant criterion is not applied for the impeller due to its complex shape and relatively coarse mesh. It is meshed with Patch Conforming tetrahedron elements.
d) Constraint:
   1. Initially to validate the Torsional theoretical analysis, the upper surface of the shaft is kept free to rotate in Z-direction and all other degrees of freedom are constrained.
   2. Then to validate the Transverse theoretical analysis, the upper surface of the motor shaft is assigned a fixed support (i.e. all six DOF are constrained).
e) Solution: Twelve modes are selected to be extracted for each of the analyses.

The top shaft is held up by the axial thrust bearing placed in the motor stool. This bearing takes up the complete load of the rotor assembly. Hence, this thrust bearing is modelled as the fixed support for the rotor assembly. The upper surface of the top shaft is initially constrained in all but rotational Y direction; Y axis being the axis of the shaft. This is to determine the torsional frequencies for validating against those obtained from Holzer's method. Then, the upper surface of the top shaft is constrained in all directions, i.e. all degrees of freedom (DOFs) are set to zero. This is to determine the transverse frequencies.

B. Results

The natural frequencies obtained from ANSYS Workbench
are summarized in the table below.

TABLE IIIII
COMPARISON OF THEORETICAL AND CORRESPONDING NUMERICAL NATURAL FREQUENCIES

Mode	Torsional Theoretical Analysis	Numerical I
	Frequency [Hz]	Frequency [Hz]
1.	2	2
2.	7	7
3.	10	9
4.	15	16
5.	17	17

Mode	Transverse Theoretical Analysis	Numerical II
	Frequency [Hz]	Frequency [Hz]
1.	1	1
2.	6	7
3.	13	12
4.	23	22
5.	36	40
6.	50	57

As seen above, there exist some discrepancies with the theoretical and numerical results. These discrepancies can be accounted for errors due to non-linearities and inability to perfectly model the system as per real life conditions.

Regardless of those discrepancies, the fundamental frequencies in all the analyses agree with each other within 10% of error. According to the reference [2] and company's guidelines, the numerical model can be considered to be validated.

C. Validating with Experimentation Phase-One

The experimental setup available can be modelled with its boundary conditions in ANSYS for predicting its natural frequencies. The setup available for experimentation is described below.

The complete rotor assembly is erected on the ground. The top shaft was lifted by a crane and steel rope. As a result, the shaft was now free to rotate in the bearing supports enclosed in the shaft tubes. This setup corresponds to a free-free condition of the system supported intermediately by four bearings.

The same setup is modelled into ANSYS. A beam with a body-to-ground connection is used to model the steel wire rope. The self weight of the system also needs to be considered. For this an acceleration of 9.81 m/s^2 (i.e. 9810 mm/s^2) needs to be applied in the vertically upward direction. Acceleration in the upward direction causes the inertia of the system to act vertically downward. This is similar to the weight of the system acting in the real environment. However, acceleration cannot be applied in a modal analysis. Hence, a static structural analysis with the above mentioned boundary conditions is performed. The results of this analysis are fed to the modal analysis as 'Pre-stress'. This modal analysis is now solved to determine the natural frequencies of the system in the working environment. The results of this analysis are mentioned later in the experimentation section.

D. Numerical Analysis of Actual System

Natural frequencies of a dynamic system are boundary condition dependent. The natural frequencies derived so far correspond to the boundary conditions assumed as per the theoretical analyses. However, the system will behave differently with actual boundary conditions. Hence, after the validation using theoretical analyses, real-life boundary conditions can be applied on the system to derive its natural frequencies in the working environment. The constraints and boundary conditions in the working environment are described as follows.

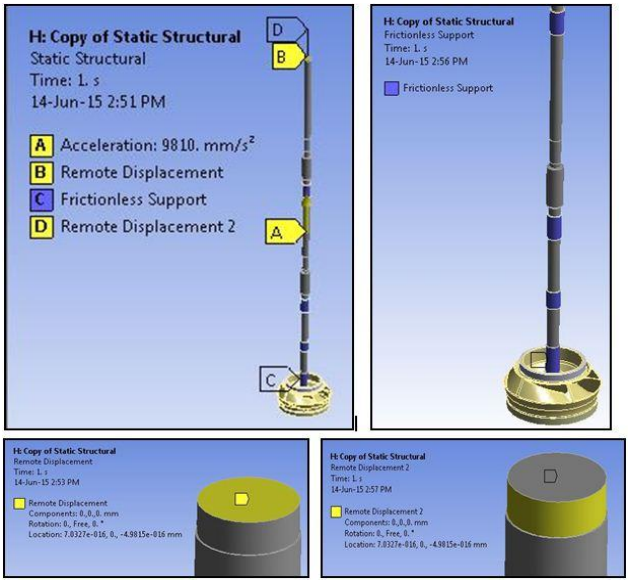
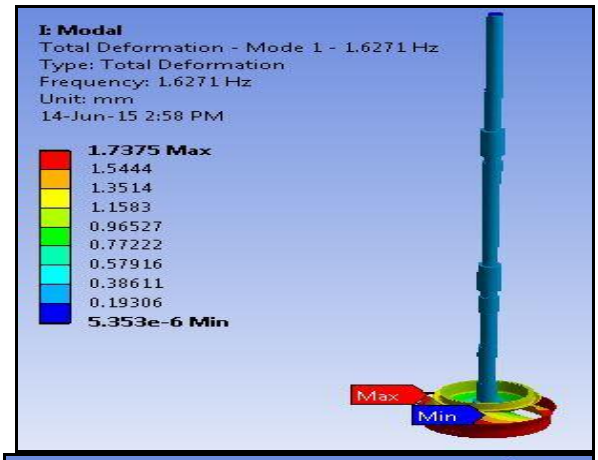


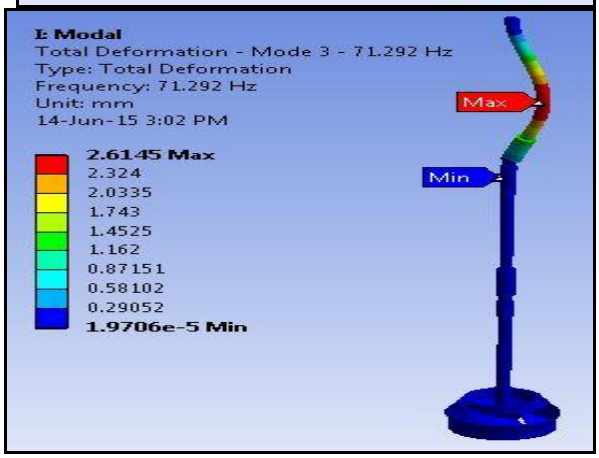
Fig.8 Boundary Conditions in Actual System

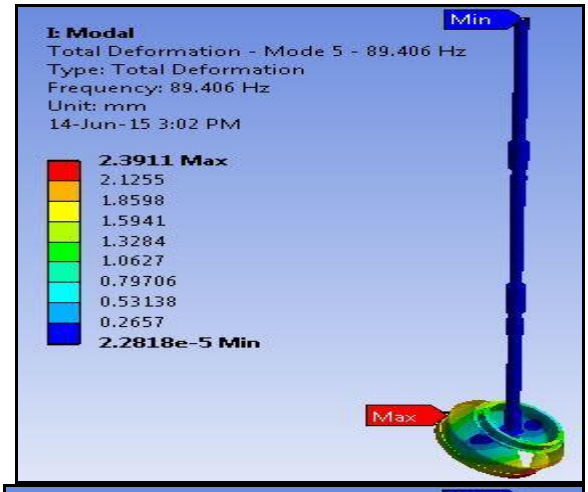
The top shaft of the rotor assembly is held firmly in the thrust bearing. The gland packing, provided to stop the water coming out of the delivery bend, also provides support for the top shaft. The intermediate shaft is enclosed in a shaft tube. This shaft tube holds a thordon bearing support. The pump shaft has three thordon bearing supports, viz; one in the shaft tube enclosing it and two in the diffuser.

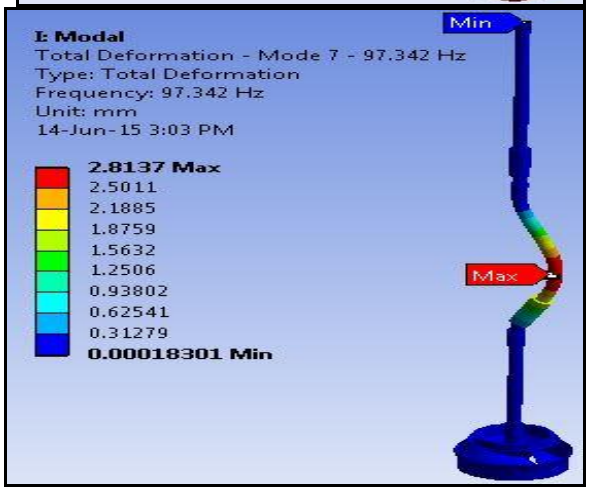
Remote Displacement is applied to the top surface of the top shaft constraining all degrees of freedom but rotational (along the axis of the shafts). Also, Remote Displacement 2 is applied on a circumferential surface to model the holding of the top shaft by the thrust rollers. An acceleration of

9810mm/s^2 is applied in the vertical direction. Rest all procedure is same as mentioned in the previous section. The results of this analysis are presented below,









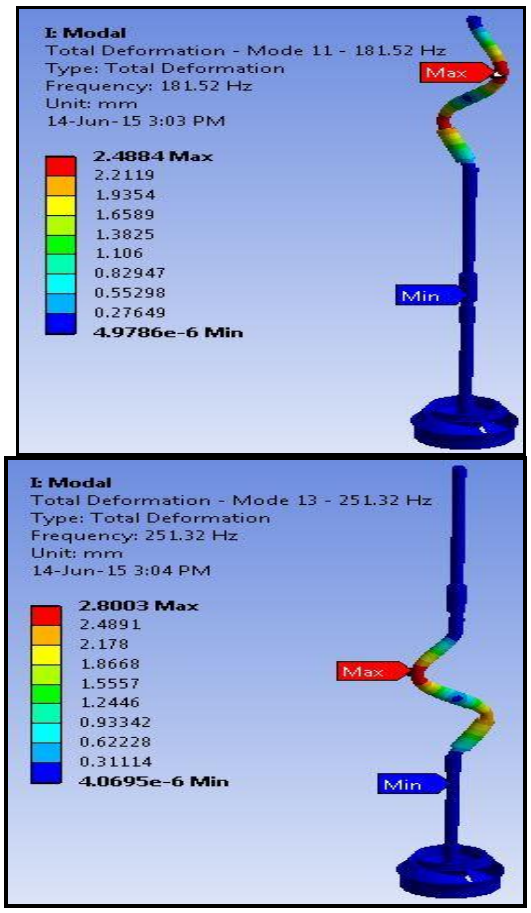


Fig.9 Mode Shapes in Actual System

TABLE IVV
NUMERICAL NATURAL FREQUENCIES OF ACTUAL SYSTEM

Sr. No.	Natural Frequency (Hz)
1	2
2	71
3	89
4	97
5	100
6	181

From the obtained results it is clear that, no natural frequency of the system lie within ±20% of its forcing frequency. Hence, the system will not be subjected to higher amplitudes of vibration due to resonance.

V. EXPERIMENTATION

Experimentation is performed on the actual system through which the natural frequencies of the system in the real life environment are obtained.

A. Phase One: Determining Natural Frequencies

The system modelled in ANSYS in numerical analysis is validated with the experimentation on the actual system. The experimentation is done by detecting the structure's response to an impact applied to it. Huge structures like these need a considerable impact as input. It may harm the structure too. Hence, a bump test is performed on the system.

A bump test does not measure the excitation. It just records the response of the system to any excitation applied to the system. This experimentation requires an accelerometer and software to read the response measured by the accelerometer only. As a result, only the values of the natural frequencies of the system are obtained. The mode shapes corresponding to these frequencies cannot be achieved through this experimentation.

The results of the experimentation carried out by the company are compared with the numerical analysis.

TABLE V
COMPARISON OF EXPERIMENTAL AND CORRESPONDING NUMERICAL NATURAL FREQUENCIES

Mode	Experimental Analysis	Numerical Analysis
	Frequency [Hz]	Frequency [Hz]
1.	11	12
2.	56	61
3.	100	97
4.	150	149
5.	250	251

The comparison of the results above validates the numerical model being used. The results obtained from the numerical analyses can thus be taken to be in close approximation with the actual results.

B. Phase-Two: Determining Vibration Amplitude

The numerical analysis performed with actual boundary conditions conform that, no natural frequency of the system lie within ±20% of its forcing frequency. As a result of this, the objective of keeping the vibrational amplitudes of the system within those specified in ISO 10816 will be satisfied. This is validated by measuring the response of the system in the working condition. A separate experimentation is carried out on the complete assembled VT pump by the company.

The vibration amplitudes observed are less than those specified in the standard. Also, the natural frequencies observed and those obtained through similar numerical model previously are in close approximation with each other; within 10% error margin. This validates the numerical validation with the experimental phase-two. Hence, it can be concluded that, the design is safe and would not resonate after being installed on-site.

VI. CONCLUSION

The rotor assembly is modelled as a torsional spring-mass system and continuous cantilever shaft with mass at free end. The system is analyzed theoretically, numerically and experimentally. Following conclusions can thus be inferred,

- The natural frequencies and their corresponding mode shapes are within ±10% of the results of the similar numerical model analyzed.
- In the numerical analysis performed with actual boundary conditions, no natural frequency of the system lie within ±20% of its forcing frequency.
- Experimentation on the Actual system conforms that the systems' vibration amplitude is within the vibrational limits specified in ISO 10816
- The system does not resonate at operational frequency

ACKNOWLEDGMENT

The authors are grateful to Dr. Ashok Ramteke of WILO ARTEC and the entire R&D team of WILO Mather Platt Pumps Pvt. Ltd. for their valuable guidance and encouragement throughout the course of the work.

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